Error

*This accident was caused by an error. The term **error** is used in many different ways. We will define error as the difference between a measured value and a known value.*

The word error is used to mean many different things and its use in science can sometimes be confusing. The term *error* can be used to mean *mistake*, or the *difference between two values*. Sometimes the words *error* and *uncertainty* are used interchangeably.

We'll use the term ***error*** to mean the difference between a measured value and the true or actual value. This means we identify an error as occuring when the experimentally determined number is not the same as an actual, or true number.

Error can be expressed as the difference between a measurement and the actual value, called ***absolute error***. Error is usually expressed as a positive value, so we use the absolute value of the difference:

$$\text{absolute error} = |\text{measurement} - \text{actual value}|$$

Error can be expressed as a percent of the actual or real value, called ***relative error***. Relative error is calculated like this:

$$\text{relative error} = \frac{|\text{measurement} - \text{actual value}|}{\text{actual value}} \times 100$$

Note that the concept of error as we use it here makes sense only in cases where an actual value is known exactly, such as when we count the number of people in a room. For measured values, as we will see in the next section, there is no known value so we can't use this method.

We'll use the term ***accuracy*** to mean *the amount of relative error in a measurement*. The accuracy of a measurement is how close it is to the actual value.

Question Set 1.0: Error and Percent Error

1. *An average chicken egg has a mass of 50 grams. You weigh a bag of eggs and find a mass of 1840 grams.*
   a. *What is the most likely number of eggs in the bag?*

   b. *Now you carefully count the eggs and find 39 eggs. What is the percent error of your predicted number of eggs?*

2. *Greek philosopher/scientist Eratosthenes measured the circumference of the earth in the year 240 BC (1732 years before Columbus sailed). His equipment was: a hole in the ground, shadow made by sunlight, and very keen reasoning. His results were amazingly accurate. In his calculations, he used a unit of distance called a* ***stadia****. Since no one today is exactly sure how long the stadia is, there is some controversy about how accurate Eratosthenes's results are.*

   a. *If we assume that Eratosthenes used the most common unit for stadia, then his measurement for the earth's circumference (converted to kilometers) is 46,620 km. An accepted value for the average circumference of the earth is 40,041.47 km. What is the percent error between Eratosthenes's measurement and the accepted value?*

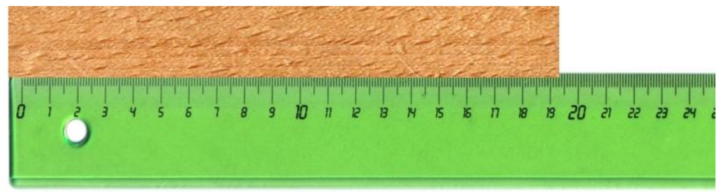
   b. *If we assume that he used a less common "Egyptian Stadium" as his unit for length, his result would be 39,690 km. What, in this case, would be the percent error between Eratosthenes's measurement and the accepted value?*

1.1 Sources of error: ***Systematic Error***

One type of error that can affect measurements is called **systematic error**. Systematic errors are ones that consistently cause the measurement value to be either too large or too small. Systematic errors can be caused by faulty equipment such as mis-calibrated balances, or inaccurate meter sticks or stopwatches. For example, if a scale is calibrated so that it reads 5 grams when nothing is on the tray, then all the readings taken with that scale will be 5 grams higher than they should be. This is an example of a systematic error that always causes masses measured with that scale to be too high.

Other systematic errors occur when equipment is used incorrectly, like reading from the wrong end of the meter stick, or forgetting to subtract the weight of the container when finding the mass of a liquid, or converting units incorrectly.

How long is the block of wood pictured at right?
Notice that the wood is lined up with the left end of the ruler, but the zero mark is not at the left end. All measurements done this way would yield measured values that are less than the true length. This is an example of systematic error.



Often systematic errors can be eliminated if you know they exist. For example, if you discover that the balance you used in a lab showed a reading of 5 grams when the tray was empty, you could go back and subtract 5 grams from all your values to reduce error in your result. In some cases, it can be very difficult to identify systematic errors. In these cases, the errors often go undiscovered until another measurement is made using a different measuring technique.

We can never be sure that our experiments are completely free from systematic errors. The best way to add confidence to our measurements is to devise an experiment to measure the same quantity by a completely different method that is unlikely to have the same error. If our new technique produces different results, one or both experiments may suffer from unidentified systematic errors. Another guide to systemic errors is the absolute or percent error in the result of the experiment. Systematic errors create more inaccuracy than other types of error

1.2 Sources of error: ***Random*** Error

Let's consider another source of error in the times recorded in arunning race. No matter how careful the timer was, they would never be able to stop the watch at the exact instant the runner crossed the line. Sometimes they would stop the watch slightly before the runner reached the line, sometimes slightly after. This is an example of a source of random error. Random error is when variations in the measurements occur without a predictable pattern. If repeated measurements are made, random errors cause the measured value to vary, sometime above and below the actual measured value. Because of this, random error causes uncertainty in measurements.

We can determine how much random error our measurements have by repeating the measurements many times. If our results are identical or nearly the same, with a very low uncertainty, this indicates a small amount of random error. If, on the other hand, our results are different each time we measure the same thing, we must have random error affecting the results.

Random errors can be reduced, but never eliminated. Random error does not always prevent our measurements from being useful, but it does contribute to uncertainty - so uncertainty in your results is often a guide to the random error that occured in the experiment.

Question Set 1.2: Systematic and Random Error

1. *Consider an experiment to determine the average acceleration of a ball dropped from a height of 1 meter. Students stand a meter stick on a table top and use a stopwatch to measure the time for the ball to fall from the top of a meter stick to the table. One student drops the ball and another student watches and carefully starts the watch*
   a. *Identify three possible sources of systematic error:*
      i.
      ii.
      iii.
   b. *Identify three possible sources of random error:*
      i.
      ii.
      iii.
2. *Make two suggestions for how the students could change their experiment to improve their results. State whether your suggestion would reduce systematic or random error*
3. *In some cases, systematic error can be difficult or impossible to identify. For example, the balance you use in lab might be damaged in such as way that it causes all masses less than 100 grams to seem 50 grams lighter than they are. How, then, can you provide evidence that your measurements do not have systematic error?*
4. *Random errors are often easy to identify, but impossible to eliminate. How can you determine whether your measurements contain random error?*